Uniform Damping Control of Spacecraft

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This paper introduces the uniform damping control of flexible spacecraft. The dynamic characteristics of spacecraft are reviewed and a criterion for dynamic performance is described by a uniform damping control approach which exhibits three distinctly attractive features. It is shown that 1) the associated uniform damping control law is independent of the spacecraft stiffness, 2) the associated control forces are proportional to the spacecraft mass density, and 3) the uniform damping control law is decentralized. The uniform damping control solution is shown to represent a first-order approximation to a special globally optimal control problem. Also, the implementation of uniform damping control is considered using discrete (in space) actuation and sensing type devices. Robustness in the presence of errors due to implementing the control using discrete components is characterized.

Introduction

OVER one hundred and fifty years ago, investigations into the behavior of beams and membrances in bending vibration, rods and shafts in torsional vibration, and bars in longitudinal vibration were conducted. Over the past one hundred years, considerable effort has been directed toward extending these classical results to structures of arbitrary shape and under the combined effects of bending, torsion, and longitudinal vibration. The extension of these classical works to the so-called field problems together with the development of numerical methods of solution for field problems has brought about the modern field of structural dynamics. More recently, with assistance from classical variational methods and modern linear system concepts, the dual theories of linear optimal control and estimation were formulated along with modern control methods and strategies. Based on these developments, the control of flexible spacecraft has been set on a firm mathematical foundation.

To bridge the gap between engineering design and the mathematical foundations upon which the control of spacecraft rests, a variety of concepts have been introduced. The representation of the motion of a spacecraft (or any structure) as a series of natural motions together with the representation of the forces acting on a spacecraft as a series of natural forces led to the first methods of modal control.[1,2] The phenomena of control spillover and observation spillover were introduced[3,4] and the method of independent modal-space control (IMSC)[5-7] was presented together with the new concept of simultaneous internal and external decoupling. A number of examinations of the robustness of modal control methods were then conducted,[8-11] and, to improve implementation capabilities, decentralized controls were characterized.[12,13] Finite-dimensional approximations to distributed controls were investigated,[14,15] and, to improve dynamic performance, globally optimal controls were characterized.[15]

As with any engineering design process, the design of a spacecraft vibration suppression system can be carried out in two independent steps. The first step consists of identifying the "best" performance. Because the state of a spacecraft is distributed over its domain, achieving the best dynamic performance will require distributed actuation and sensing devices where it is recognized that the use of these distributed devices is for the most part impractical. The second step consists of constructing a control system of minimal cost which exhibits dynamic performance that is as close to the best dynamic performance as possible. Therefore, the second step consists of implementing the control obtained in the first step using discrete actuation and sensing devices, with consideration given to robustness and reliability issues.

In the next section, the dynamics of flexible spacecraft are reviewed. The best dynamic performance, as described later, is achieved by a uniform damping control. It will be shown that uniform damping control represents a first-order approximation to a special globally optimal control problem. In addition, the implementation of uniform damping control using discrete actuators and sensors will be presented followed by an examination of the robustness of uniform damping control in the presence of errors due to implementing the control using discrete components. Finally, a numerical example will be illustrated.

The Dynamics of Flexible Spacecraft

A flexible spacecraft is comprised of interconnected point masses, which act upon each other through elastic restoring forces. Each point P in the domain D of the spacecraft is displaced by an amount $\boldsymbol{u}(P,t)$ at time t. The swinggle refers to vectors in a three-dimensional inertial space. The elastic restoring forces $\boldsymbol{f}_e(P,t)$ are generally a function of the displacement and the spatial derivatives of the displacement and are expressed symbolically in the form $\boldsymbol{f}_e(P,t) = -L\boldsymbol{u}(P,t)$ in which L represents an appropriate differential operator.† Assuming that the only external forces acting on the system are control forces, denoted by $\boldsymbol{f}(P,t)$, Newton's laws of motion lead to the equations of motion of the spacecraft at each point P,

$$\rho(P)\ddot{\boldsymbol{u}}(P,t) = -L\boldsymbol{u}(P,t) + \boldsymbol{f}(P,t) \tag{1}$$

in which $\rho(P)$ denotes the mass density of point P and overdots represent differentiations with respect to time.

It proves attractive to view the displacement $\boldsymbol{u}(P,t)$ of the spacecraft as an infinite sum of the so-called natural displacements $\boldsymbol{u}_r(P,t)$, written

$$\boldsymbol{u}(P,t) = \boldsymbol{u}_1(P,t) + \boldsymbol{u}_2(P,t) + \ldots$$
$$\boldsymbol{u}_r(P,t) = \boldsymbol{\phi}_r(P)u_r(t), \qquad (r=1,2,\ldots) \tag{2}$$

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†The usual assumption is made that the free motion of the spacecraft is undamped and linear, in which case the stiffness operator L is positive semidefinite, self-adjoint, and admits a discrete spectrum.

in which the natural motion $u_r(P,t)$ has shape (spatial-dependence) $\phi_r(P)$, called the natural mode of vibration, and amplitude (time-dependence) $u_r(t)$, called the modal displacement. Just as the displacement was expressed as an infinite sum, so too can the control force $f(P,t)$ be expressed as an infinite sum of natural control forces $f_r(P,t)$, written

$$f(P,t) = f_1(P,t) + f_2(P,t) + \ldots,$$

$$f_r(P,t) = \rho(P)\phi_r(P)f_r(t), \quad (r=1,2,\ldots) \quad (3)$$

in which the natural force $f_r(P,t)$ has shape (spatial-dependence) $\rho(P)\phi_r(P)$, called the natural mode of control, and amplitude (time-dependence) $f_r(t)$, called the modal control force. The natural modes of vibration can satisfy the orthonormality conditions

$$\int_D \rho(P)\phi_r^T(P)\phi_s(P)\mathrm{d}D = \delta_{rs}$$

$$\int_D \phi_r(P)L\phi_s(P)\mathrm{d}D = \omega_r^2\delta_{rs} \quad (4)$$

in which δ_{rs} is the Kronecker-delta and ω_r denotes the natural frequency of the rth natural motion. Substitution of natural decompositions, Eqs. (2) and (3), into the equations of motion and full consideration of the orthonormality conditions [Eq. (4)] leads to the natural equations of motion for the spacecraft,

$$\ddot{u}_r(P,t) = -\omega_r^2 u_r(P,t) + f_r(P,t), \quad (r=1,2\ldots) \quad (5)$$

Equation (5) represents a series of independent, second-order, ordinary differential equations. The separation of the dynamics of the spacecraft into independent equations, as described here, is called a natural decomposition and the corresponding quantities are referred to as natural quantities because this decomposition is unique for a given spacecraft. Note that the rth natural control force $f_r(P,t)$ is capable of exciting (and controlling) the sth natural motion, $u_s(P,t)$ and $\dot{u}_s(P,t)$, only for identical r and s.

Uniform Damping Control

The active control of flexible spacecraft involves the creation of control forces external to the spacecraft for the purposes of either maintaining the spacecraft at a given equilibrium state, called *vibration suppression,* or for the purposes of moving the spacecraft from one equilibrium state to another, called *maneuver.* Further discussions here address the former. Vibration suppression can be achieved by designing control forces $f(P,t)$. These forces are either explicitly a function of time called *open-loop control* or only implicitly a function of time and explicitly a function of the state of the spacecraft, called *feedback control* or *closed-loop control.* In the latter case, we say that the control forces are governed by the control law, expressed functionally as

$$f(P,t) = f[u(P,t), \dot{u}(P,t)] \quad (6)$$

When the control law is linear, Eq. (6) will have the form

$$f(P,t) = Gu(P,t) + H\dot{u}(P,t) \quad (7)$$

in which G and H are the linear control gain matrix operators to be determined by the designer. Depending on the operators G and H, $f(P,t)$ can represent distributed forces or discrete (in space) forces, and the measurements of $u(P,t)$ and $\dot{u}(P,t)$ can be distributed or discrete (in space).

Also depending on the operators G and H, the system dynamics will either improve or be expected to improve in some sense. It is this "sense" or the exact way in which the system dynamics will improve that is often times unclear and thus deserves closer attention. To this end, the control law, Eq. (7), is substituted into the equation of motion, Eq. (1), and the equations of motion for the controlled spacecraft are obtained

$$\rho(P)\ddot{u}(P,t) - H\dot{u}(P,t) + (L-G)u(P,t) = 0 \quad (8)$$

The displacement can be expressed as a linear combination of complex closed-loop modes $\psi_r(P)$ associated with the controlled spacecraft, in the form

$$u(P,t) = \psi_1(P)C_1e^{\lambda_1 t} + \psi_2(P)C_2e^{\lambda_2 t} + \ldots \quad (9)$$

in which C_r are complex constants depending on the initial conditions and $\lambda_r = -\alpha_r + i\beta_r$ are called closed-loop eigenvalues. The real parts α_r represent exponential decay rates, and the imaginary parts β_r represent frequencies of oscillation of the controlled spacecraft. It is an immediate objective to examine the motion at any given point P^* on the spacecraft. In terms of the dynamic performance, it is desirable, regardless of the initial conditions, for the motion at P^* to be asymptotically stable, so that

$$\|u(P^*,t)\| \leq U_0e^{-\alpha t} \quad (10)$$

implying that the magnitude of the motion is bounded initially by some value U_0 and decays at the exponential rate α, in which

$$U_0 = |\psi_1(P^*)C_1| + |\psi_2(P^*)C_2| + \ldots \quad (11)$$

The satisfaction of Eq. (10) is herein defined as rendering the "best" dynamic performance. By comparing the representation of the motion of the controlled spacecraft [Eq. (9)] with that desired [Eq. (10)], it is found that the desired motion can only be arrived at if the motion at P^*, due to each and every closed-loop mode, decays at an exponential rate α_r not less than the desirable decay rate α,

$$\alpha_r \geq \alpha, \quad (r=1,2,\ldots) \quad (12)$$

It also follows from Eq. (12) that it is not necessary to dampen the motion of any of the modes at an exponential rate greater than α. In fact, any attempt to do so could require additional control effort. Therefore, Eq. (10) implies that *the best dynamic performance is satisfied by a control which dampens the motion due to each and every mode at a uniform exponential rate.* Furthermore, because Eq. (12) must be satisfied regardless of the point P^* chosen, it follows that the motion of point P^* will decay at the exponential rate α only if the motion of every other point P on the spacecraft decays at the exponential rate α. Therefore, *the desire to suppress the vibration of any single point P^* according to Eq. (10) implies that the vibration must be suppressed over the entire spacecraft at the same decay rate.* In this case, it is assumed that the modes are coupled by the spacecraft structure. Otherwise, the spacecraft motion can be represented as two independent motions, where the independent motions appear either in perpendicular directions or over separate regions of the spacecraft.

Two points remain regarding the advantages that may be derived from suppressing vibration: 1) the natural modes of vibratrion of the uncontrolled spacecraft are altered or 2) the natural frequencies of the uncontrolled spacecraft are altered. With respect to the first point, note that it has been verified that appreciable changes in the shape of the natural modes of vibration require excessively large control forces.[17-19] Therefore, it is not an objective here to alter the natural modes of the uncontrolled spacecraft. Regarding the second point, appreciable changes in the natural frequencies of the uncontrolled spacecraft are achieved by persistently applying relatively large control forces. Therefore, it is also not an objective to alter the natural frequencies.

In view of these remarks, the best dynamic performance is characterized by a uniform damping control in which 1) the motion due to each and every mode decays at a single exponential rate, 2) the controlled frequencies of oscillation are identical to the uncontrolled natural frequencies, and 3) the closed-loop modes of vibration are identical to the uncontrolled natural modes of vibration. By examining the natural equations of motion for the spacecraft [Eq. (5)] and the natural decompositions [Eqs. (2) and (3)], the uniform damping control law for each natural motion is obtained

$$f_r(t) = g_r u_r(t) + h_r \dot{u}_r(t)$$
$$g_r = -\alpha^2, \qquad h_r = -2\alpha \qquad (r = 1,2,...) \tag{13}$$

Substituting the uniform damping control laws‡ for each natural motion [Eq. (13)] into the natural decomposition of the control force [Eq. (3)], the following is obtained

$$\begin{aligned} f(P,t) &= \rho(P)\phi_1(P)[-2\alpha\dot{u}_1(t) - \alpha^2 u_1(t)] \\ &\quad + \rho(P)\phi_2(P)[-2\alpha\dot{u}_2(t) - \alpha^2 u_2(t)] + ... \\ &= -2\alpha\rho(P)[\phi_1(P)\dot{u}_1(t) + \phi_2(P)\dot{u}_2(t) + ...] \\ &\quad - \alpha^2\rho(P)[\phi_1(P)u_1(t) + \phi_2(P)u_2(t) + ...] \end{aligned} \tag{14}$$

Substituting the natural decomposition for Eq. (2) into Eq. (14), the uniform damping control law is obtained

$$f(P,t) = -2\alpha\rho(P)\dot{u}(P,t) - \alpha^2\rho(P)u(P,t) \tag{15}$$

Upon examining the uniform damping control law [Eq. (15)], three characteristics are observed: 1) The uniform damping control law is independent of the spacecraft stiffness; 2) The control forces are proportional to the mass density $\rho(P)$; and 3) The control forces are decentralized, i.e., the control force $f(P,t)$ depends on $u(P^*,t)$ and $\dot{u}(P^*,t)$ only for identical points, $P = P^*$.

The first characteristic implies that the "best" dynamic performance can be implemented without explicit knowledge of the spacecraft stiffness. The second characteristic should support one's intuition. Indeed, as the mass density increases, the control force will also increase proportionally. The third characteristic will suggest a simple and reliable method of implementation.

The Globally Optimal Solution

With the development of linear optimal control methods, various quadratic functionals have been minimized for the purposes of computing control laws in which an adequate dynamic performance is obtained with minimal control effort. The quadratic functional commonly consists of the sum of two terms, with one term representing an energy that reflects a dynamic performance of the spacecraft and the other term representing a control effort that reflects a cost in fuel. Usually the energy and control effort have different units in which case the choice to minimize the sum of the two terms seems irrationally based and leads to a solution that is difficult to interpret. Nevertheless, the approach is quite powerful since it provides a method for computing control gains G and H.

In this section, the minimization of a particular quadratic functional leads to a uniform damping control, and an interpretation of the quadratic functional is rendered. In particular, we define the globally§ optimal control problem with associated quadratic functional

$$J = \int_0^\infty \left[E(t) + \frac{1}{2\alpha^{*2}}\int_D f^T(P,t)f(P,t)/\rho(P)\mathrm{d}D\right]\mathrm{d}t \tag{16}$$

in which $E(t)$ represents the total energy of the spacecraft, the right term in the integrand represents the control effort, and α^* is a positive number chosen by the designer.

The solution to the globally optimal control problem is unique and the control gain operators in Eq. (7) were found in the closed-form[15] as follows

$$G = \rho(P)\sum_{r=1}^{\infty}\phi_r(P)g_r\int_D \rho(P)\phi_r^T(P)(\ )\mathrm{d}D$$
$$H = \rho(P)\sum_{r=1}^{\infty}\phi_r(P)h_r\int_D \rho(P)\phi_r^T(P)(\ )\mathrm{d}D \tag{17}$$

in which

$$g_r = \omega_r^2 - \omega_r(\omega_r^2 + 2\alpha^{*2})^{1/2}$$

and

$$h_r = -[-2\omega_r^2 + 2\alpha^{*2} + 2\omega_r(\omega_r^2 + 2\alpha^{*2})^{1/2}]^{1/2}, \qquad (r = 1,2,...) \tag{18}$$

G and H are self-adjoint, possess eigenfunctions identical to the natural modes of vibration and possess associated eigenvalues $g_r (r=1,2,...)$ and $h_r (r=1,2,...)$, respectively. It was shown in Ref. 15 that globally optimal control is a natural control. (Natural controls are characterized by the preservation of the natural characteristics of the system. The natural coordinates as well as the natural modes of vibration are preserved during the control action. Natural controls may be obtained using the independent modal space control (IMSC) method.)

The natural motions for the spacecraft include three translational motions and three rotational motions called rigid-body motions. The natural frequencies for rigid-body motions are zero and from Eq. (18), it is seen that $g_r = 0$ for rigid-body motions. Therefore, the velocity of the rigid-body motion is driven to zero, though the displacement is driven to any nonzero position in space. This can be expected because the potential energy of a rigid-body displacement is zero and, thus, not minimized by the quadratic functional [Eq. (16)]. Next, the elastic natural motions with associated non-zero natural frequencies will be examined. Consider the single term Taylor series expansions of g_r and $h_r (r=1,2,...)$ in Eq. (18) in the form

$$[\omega_r^2 + 2\alpha^{*2}]^{1/2} = \omega_r + \alpha^{*2}/\omega_r$$
$$\alpha^* < \omega_r \qquad (r = 1,2,...) \tag{19}$$

Substitution of Eq. (19) into the expansions for the globally optimal control gain eigenvalues [Eq. (18)], yields

$$g_r = -\alpha^{*2}, \qquad h_r = -2\alpha^*, \qquad (r = 1,2,...) \tag{20}$$

When comparing the control gain eigenvalues [Eq. (20)] with those associated with uniform damping control [Eq. (13)], the parameter α^* chosen in the quadratic functional [Eq.

‡The constants g_r and h_r are in fact the control gain eigenvalues of G and H, respectively.[15]

§The term "global" refers here to a minimization subject to no active constraints other than the equations of motion. Nonglobally optimal control problems constrain the control force to a proper subset of the linear space spanned by the natural modes of vibration. For example, the assumption that the control forces are discrete (in space) represents a constraint and leads to a nonglobally optimal solution.

(16)] is found to be identical to the uniform decay rate α, i.e.,

$$\alpha^* = \alpha \tag{21}$$

so that uniform damping control of the elastic motion at an exponential rate α represents a first-order approximation to globally optimal control. The parameter α^* can now have units of rad/s so that the two terms in the integrand of the quadratic functional possess identical units, and the choice to minimize the sum of these two terms becomes a rational one. Also note that the second term in the integrand is inversely proportional to the mass density $\rho(P)$ so that the uniform damping control force is directly proportional to the mass density $\rho(P)$.

Implementation of Uniform Damping Control

The significant part of the elastic motion for a spacecraft is due to the natural motions having lower natural frequencies. At the same time, it is more difficult to excite (and control) the natural motions with higher natural frequencies. Also, the modal control forces $f_r(t)$ are first integrals of the control force $f(P,t)$. It follows that the distributed control law can be approximated using discrete (in space) control laws in which the control forces are discrete and the measurements of the state are discrete. It also follows that the anticipated errors due to discretization of the distributed control force will carry over to those natural motions with higher natural frequencies. In view of these remarks, a series of control laws is chosen which converges to uniform damping control and which provides as good an approximation as is possible given the limitations of the number of control forces available.

Perhaps the most intuitive approximation is one in which control forces $F_r(t)$ are placed in various regions R_r $(r=1,2,\ldots,n)$ of the spacecraft. Each control force is located at point P_r in region R_r, and is proportional to the mass M_r in its region. Denoting the measurements of displacement and velocity at points P_r by $U_r(t)$ and $\dot{U}_r(t)$, respectively, the uniform damping control laws [Eq. (15)] discretized for regions R_r are considered

$$F_r(t) = -\alpha^2 M_r U_r(t) - 2\alpha M_r \dot{U}_r(t), \quad (r=1,2,\ldots,n) \tag{22}$$

The discrete uniform damping control law [Eq. (22)] converges to the distributed uniform damping control [Eq. (15)]. In order to demonstrate this, let the discrete control forces be represented by the distributed control force, in the form

$$f(P,t) = F_1(t)\delta(P-P_1) + F_2(t)\delta(P-P_2) + \ldots + F_n(t)\delta(P-P_n) \tag{23}$$

in which $\delta(P-P_r)$ is the Dirac-delta function and note that

$$U_r(t) = u(P_r,t), \quad \dot{U}_r(t) = \dot{u}(P_r,t) \qquad (r=1,2,\ldots,n) \tag{24}$$

and

$$M_r = \int_{R_r} \rho(P)\mathrm{d}D \qquad (r=1,2,\ldots,n) \tag{25}$$

When considering Eqs. (23-25) and comparing the discrete uniform damping control law [Eq. (22)] with the general form [Eq. (7)], the associated control gain operators in closed-form are obtained

$$G_n = -\alpha^2 \sum_{r=1}^{n} M_r \delta(P-P_r), \quad H_n = -2\alpha \sum_{r=1}^{n} M_r \delta(P-P_r) \tag{26}$$

Clearly, G_n and H_n converge to G and H, respectively, because

$$\sum_{r=1}^{n} M_r \delta(P-P_r)$$

converges to the mass density $\rho(P)$.

Because the uniform damping control law [Eq. (22)] is decentralized, those features that are characteristic of decentralized control systems[12,13] are applicable here as well. In particular, the system stability can be guaranteed independently in each region R_r. Therefore, the system stability can be guaranteed where an actuator fails and when a sensor fails provided the actuator in the region R_r of failure is turned off.

With regard to the choice of regions and the placement of the control forces, the quality of a given choice can be examined by inspecting the deviation of the closed-loop eigenvalues associated with the discrete control law from the desired eigenvalues associated with the distributed control law. In fact, approximate eigenvalues can be obtained without solving the eigenvalue problem for the controlled spacecraft as is demonstrated in the next section.

Robustness of Uniform Damping Control

In practice, the mathematical model of a spacecraft is only approximately known. Because the distributed uniform damping control law is independent of the spacecraft stiffness, modeling errors in stiffness will not have an adverse effect on the dynamic performance of the controlled spacecraft. However, some degradation in performance is anticipated when the distributed controls are discretized. The effects arising due to discretization of the controls can be examined by examining changes in the eigenvalues of the controlled spacecraft.[20] The computation of the eigenvalues of the controlled spacecraft can be avoided by instead computing bounds for the eigenvalues of the controlled spacecraft. It is understood here that bounds for the eigenvalues provide a conservative characterization of robustness.

When substituting the discrete uniform damping control law [Eqs. (7) and (26)] into the equations of motion for the spacecraft [Eq. (1)] and considering the natural decompositions [Eqs. (2) and (3)] with full consideration of the orthonormality conditions [Eq. (4)], the modal equations of motion for the controlled spacecraft have the form

$$\ddot{u}_r(t) = -2\alpha\dot{u}_r(t) - (\alpha^2+\omega_r^2)u_r(t) + \sum_{s=1}^{\infty}[g_{rs}u_s(t) + h_{rs}\dot{u}_s(t)], \qquad (r=1,2,\ldots) \tag{27}$$

in which

$$g_{rs} = \alpha^2\left[\delta_{rs} - \sum_{t=1}^{n} M_t \phi_r^T(P_t)\phi_s(P_t)\right]$$

$$h_{rs} = 2\alpha\left[\delta_{rs} - \sum_{t=1}^{n} M_t \phi_r^T(P_t)\phi_s(P_t)\right] \quad (r,s=1,2,\ldots) \tag{28}$$

For the purposes of examining robustness, the modal Eq. (27) can be rewritten in the state space by introducing the change of complex variables

$$u_r(t) = Re\{w_r(t)\}, \; \dot{u}_r(t) = Re\{\lambda_r w_r(t)\} \quad (r=1,2,\ldots) \tag{29}$$

in which $\lambda_r = -\alpha \pm i\omega_r$ are exactly the eigenvalues that would have been obtained had the uniform damping control been distributed. Substituting the change of variables in Eq. (29) into the modal Eqs. (27), the complex modal state equations are obtained

$$\dot{w}_r(t) = \lambda_r w_r(t) + \tfrac{1}{2} \sum_{s=1}^{\infty} [\Lambda_{rs} w_s(t) + \bar{\Lambda}_{rs} \bar{w}_s(t)] \qquad (r=1,2,\ldots) \qquad (30a)$$

$$\Lambda_{rs} = (g_{rs} + \lambda_s h_{rs})/(i\omega_r) \qquad (30b)$$

Based on Gerschgorin's two theorems, bounds for the eigenvalues of the controlled spacecraft can be found.[20] The eigenvalues are each contained within the circles having centers C_r and associated radii R_r, given by

$$C_r = \lambda_r + \Lambda_{rr}/2, \qquad R_r = \sum_{\substack{s=1 \\ s \neq r}}^{\infty} |\Lambda_{rs}| \qquad (r=1,2\ldots) \qquad (31)$$

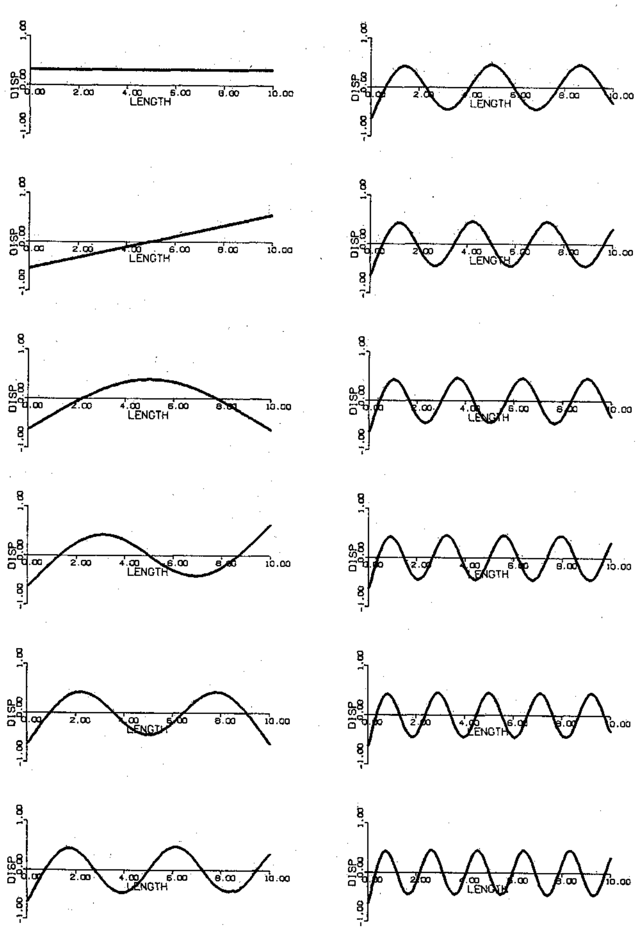

Fig. 1 Lowest 12 natural modes of vibration.

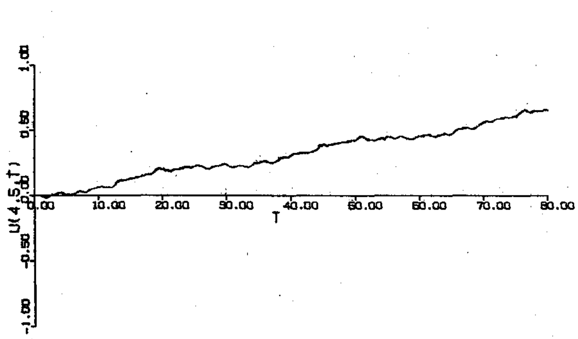

Fig. 2 Free response at $x = 4.5$ to initial impulse at $x = 8.5$.

Of course, the eigenvalues will always lie in the left-half plane of the circles because stability is guaranteed for positive gains in Eq. (22). Note that the centers C_r are first-order approximations of the closed-loop eigenvalues of the distributed uniform damping control. As the number of actuation devices increases, the circular bounds of Eq. (31) tend toward points centered at the exact eigenvalues ($R_r = 0$, $C_r = \lambda_r$). Using Eqs. (31) and (28), approximate controlled eigenvalues are computed, and the solution to the eigenvalue problem for the controlled spacecraft is avoided.

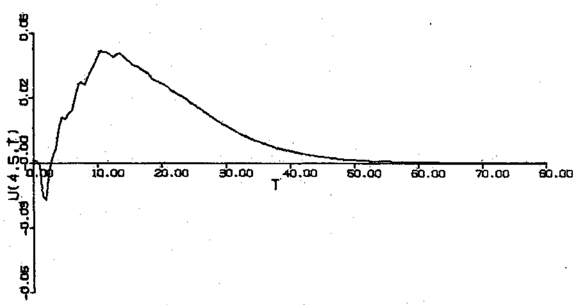

Fig. 3 Controlled response given three control forces.

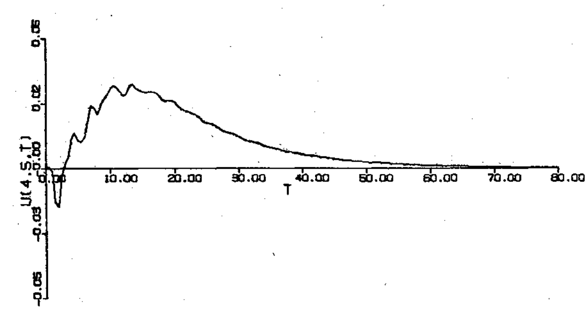

Fig. 4 Controlled response given five control forces.

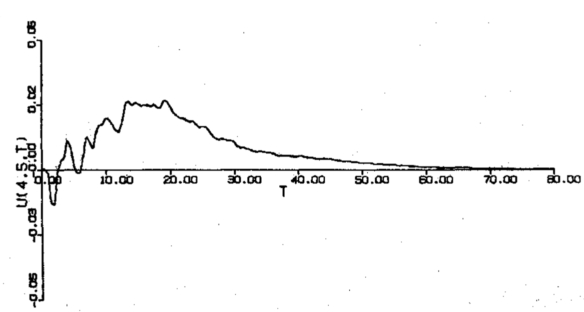

Fig. 5 Controlled response given distributed controls.

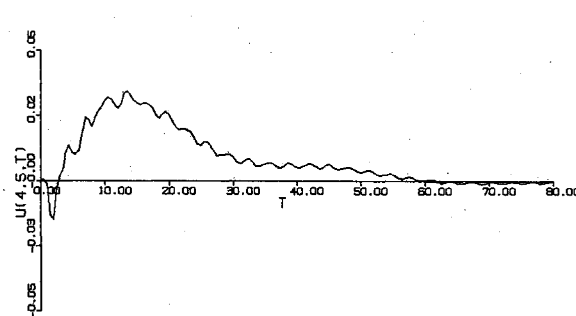

Fig. 6 Controlled response given 5 control forces in which the first control force is turned off after 10 s and the third control force is turned off after 20 s.

Table 1 Free-free beam vibration

12 lowest natural frequencies
10.88124
8.90732
7.13079
5.55165
4.16991
2.98556
1.99859
1.20903
0.61673
0.22373
0.00000
0.00000

Table 2 Case 1: controlled eigenvalues

12 lowest eigenvalues given 3 control forces	
−0.156523	±10.837580*i*
−0.226506	±8.860081*i*
−0.159518	±7.087265*i*
−0.225830	±5.494731*i*
−0.157754	±4.117811*i*
−0.222915	±2.915502*i*
−0.157720	±1.925238*i*
−0.282593	±1.056921*i*
−0.789393	±0.434386*i*
−0.670601	±0.089764*i*
−0.139146	
−0.093940	
−0.075065	
−0.058426	

Table 3 Case 2: controlled eigenvalues

12 lowest eigenvalues given 5 control forces	
−0.168262	±10.864591*i*
−0.151845	±8.890696*i*
−0.110871	±7.112176*i*
−0.210372	±5.529351*i*
−0.171788	±4.153424*i*
−0.154027	±2.964561*i*
−0.113319	±1.967720*i*
−0.255464	±1.155694*i*
−0.210279	±0.568499*i*
−0.181446	±0.141047*i*
−0.163629	
−0.083912	
−0.266055	
−0.063278	

Uniform Damping Control of a Free-Free Beam

The effectiveness of uniform damping control can be illustrated through the following numerical example in which the position, orientation, and elastic motion of a free-free beam is suppressed. It is understood that the results extend to the field problem. The beam is 10 units long with uniform mass properties $\rho(x)=1$ and uniform stiffness $L=\partial^4/\partial x^4$. The beam statisfies the boundary conditions $\partial^2 u(x,t)/\partial x^2=0$ and $\partial^3 u(x,t)/\partial x^3=0$ at both ends $x=0,10$. The natural modes of vibration of the beam admit closed-form expressions.[21] The lowest 12 natural modes of vibration are shown in Fig 1. The corresponding natural frequencies are given in Table 1. The free response at $x=4.5$ to an initial impulsive force of 0.1 units at $x=8.5$ is shown in Fig. 2. The following cases of uniform damping control of the given free-free beam at a desirable exponential rate of $\alpha=0.1$ are given.

Case 1: Three discrete control forces are located at $x_1=0$, $x_2=5$, $x_3=10$ and $M_1=M_2=M_3=3.33$ in Eq. (17). The controlled eigenvalues are given in Table 2. The controlled response is shown in Fig. 3.

Case 2: Five discrete control forces are located at $x_1=0$, $x_2=2.5$, $x_3=5$, $x_4=7.5$, $x_5=10$ and $M_r=2$ $(r=1,2,\ldots 5)$. The controlled eigenvalues are given in Table 3. The controlled response is shown in Fig. 4.

Case 3: A distributed uniform damping control is considered. The controlled eigenvalues are ideally $\lambda_r=-\alpha\pm i\omega_r$ $(r=1,2,\ldots)$. The controlled response is shown in Fig. 5.

Case 4: The second case is considered in which the first discrete control force $F_1(t)$ is turned off at $t=10$ s, and the third discrete control force is turned off at $t=20$ s. The controlled response is shown in Fig. 6.

Final Remarks

In retrospect, the research community involved in the development of control methods for flexible spacecraft focuses on a variety of issues. The relationship between these issues is sometimes unclear. Some scientists, in their attempt to remain "practical," investigate decentralized control methods. Others give special attention to centralized methods of control. Of course, this practice originated with the development of linear optimal control theory. Still others concern themselves with robustness issues, particularly because it is difficult to come up with an accurate mathematical model of structural stiffness. This difficulty in turn makes it undesirable to design a control system that relies on the fidelity of the mathematical model of structural stiffness.

Conclusions

This paper has introduced the concept of uniform damping. In doing so, it has highlighted relationships between the decentralized and centralized methods of control and the robustness issue. Following the introduction was a description of how one arrives at the objective of uniformly damping spacecraft motion. In subsequent sections, other aspects of uniform damping were described. It was shown that the uniform damping control laws are decentralized, independent of structural stiffness and proportional to the distribution of mass over the spacecraft. It was also shown that uniform damping is near-globally optimal. Finally, it was demonstrated in a numerical example how uniform damping is implemented using a relatively small number of control forces.

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